

# $\Lambda(p)$ -sets and the limit order of operator ideals

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## Abstract

Given an infinite set  $\Lambda$  of characters on a compact abelian group we show that  $\Lambda$  is a  $\Lambda(p)$ -set for all  $2 < p < \infty$  if and only if the limit order of the ideal  $\Pi_\Lambda$  of all  $\Lambda$ -summing operators coincides with that of the ideal  $\Pi_\gamma$  of all Gaussian-summing operators, i. e.  $\lambda(\Pi_\Lambda, u, v) = \lambda(\Pi_\gamma, u, v)$  for all  $1 \leq u, v \leq \infty$ . This is a natural counterpart to a recent result of Baur which says that  $\Lambda$  is a Sidon set if and only if  $\Pi_\Lambda = \Pi_\gamma$ . Furthermore, our techniques, which are mainly based on complex interpolation, lead us to exact asymptotic estimates of the Gaussian-summing norm  $\pi_\gamma(\text{id} : \mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n)$  of identities between finite-dimensional Schatten classes  $\mathcal{S}_u^n$  and  $\mathcal{S}_v^n$ ,  $1 \leq u, v \leq \infty$ .

## 1 Introduction and results

We use standard notation and notions from Banach space theory, as presented e. g. in [DJT95], [LT79] and [TJ89]. If  $E$  is a Banach space, then  $B_E$  is its (closed) unit ball and  $E'$  its dual; we consider complex Banach spaces only. As usual  $\mathcal{L}(E, F)$  denotes the Banach space of all (bounded and linear) operators from  $E$  into  $F$  endowed with the operator norm.

For an infinite orthonormal system  $B \subset L_2(\mu)$  (over some probability space  $(\Omega, \mu)$ ) an operator  $T : E \rightarrow F$  between Banach spaces  $E$  and  $F$  is said to be  $B$ -summing if there exists a constant  $c > 0$  such that for all finite sequences  $b_1, \dots, b_n$  in  $B$  and  $x_1, \dots, x_n$  in  $E$

$$\left( \int_{\Omega} \left\| \sum_{i=1}^n b_i \cdot T x_i \right\|^2 d\mu \right)^{1/2} \leq c \cdot \sup_{x' \in B_{E'}} \left( \sum_{i=1}^n |\langle x', x_i \rangle|^2 \right)^{1/2}; \quad (1.1)$$

we write  $\pi_B(T)$  for the smallest constant  $c$  satisfying (1.1). In this way we obtain the injective and maximal Banach operator ideal  $(\Pi_B, \pi_B)$ , which became of interest recently in the theses of Baur [Bau97] and Seigner [Sei95]. For a sequence of independent standard Gaussian random variables the associated Banach operator ideal  $\Pi_\gamma$  of all Gaussian-summing operators was introduced by Linde and Pietsch [LP74], and for operators acting on finite-dimensional Hilbert spaces  $\pi_\gamma$  is also known as the  $\ell$ -norm, which turned out to be important for the study of the geometry of Banach spaces (see e. g. [TJ89]).

For an infinite subset  $\Lambda$  of the character group  $\Gamma$  of some compact abelian group  $G$  (which can be viewed as an orthonormal system in  $L_2(G, m_G)$ , where  $m_G$  denotes the normalized Haar measure on  $G$ ) Baur in [Bau97, 9.5] (see also [Bau99, 4.2]) gave the following characterization:

$\Lambda$  is a Sidon set if and only if  $\Pi_\Lambda = \Pi_\gamma$ .

Recall that a subset  $\Lambda \subset \Gamma$  is said to be a *Sidon set* if there exists  $\theta > 0$  such that for all  $Q = \sum_{\gamma \in \Lambda} \alpha_\gamma \cdot \gamma \in \text{span}(\Lambda)$  we have  $\sum_{\gamma \in \Lambda} |\alpha_\gamma| \leq \theta \cdot \|Q\|_{L_\infty(G, m_G)}$ , and for  $2 < p < \infty$  it is called a  $\Lambda(p)$ -set if there exists a constant  $c > 0$  such that for all  $\lambda \in \text{span}(\Lambda)$  we have  $\|\lambda\|_{L_p(G, m_G)} \leq c \|\lambda\|_{L_2(G, m_G)}$ ; the infimum over all such constants  $c$  is denoted by  $K_p(\Lambda)$ . Pisier in [Pi78] showed that  $\Lambda$  is a Sidon set if and only if

$$\Lambda \text{ is a } \Lambda(p)\text{-set with } K_p(\Lambda) \leq \kappa \sqrt{p} \text{ for all } 2 < p < \infty \text{ and some } \kappa > 0.$$

As a natural counterpart of Baur's result we prove the following characterization of sets which are  $\Lambda(p)$ -sets for all  $2 < p < \infty$ , with no control of  $K_p(\Lambda)$  as in Pisier's characterization of Sidon sets above:

**Theorem 1.** *For every infinite subset  $\Lambda \subset \Gamma$  the following are equivalent:*

- (a)  $\Lambda$  is a  $\Lambda(p)$ -set for all  $2 < p < \infty$ .
- (b)  $\lambda(\Pi_\Lambda, u, v) = \lambda(\Pi_\gamma, u, v)$  for all  $1 \leq u, v \leq \infty$ .

Here, the limit order  $\lambda(\mathcal{A}, u, v)$  of a Banach operator ideal  $(\mathcal{A}, A)$  for  $1 \leq u, v \leq \infty$  is defined as usual (see e.g. [Pie80, 14.4]):

$$\lambda(\mathcal{A}, u, v) := \inf\{\lambda > 0 \mid \exists \rho > 0 \forall n \in \mathbb{N} : A(\text{id} : \ell_u^n \hookrightarrow \ell_v^n) \leq \rho \cdot n^\lambda\}.$$

Note that there exist sets which are  $\Lambda(p)$ -sets for all  $2 < p < \infty$  but fail to be Sidon sets (see e.g. [LR75, 5.14]). Our proof is mainly based on complex interpolation techniques, in particular on formulas for the complex interpolation of spaces of operators due to Pisier [Pi90] and Kouba [Kou91] (see also [DM99]). These techniques also yield asymptotic estimates of the Gaussian-summing norm of identities between finite-dimensional Schatten classes:

**Theorem 2.** *For  $1 \leq u, v \leq \infty$*

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \asymp \begin{cases} n^{1/2+1/v} & \text{if } 2 \leq u \leq \infty, \\ n^{1/2+\max(0, 1/2+1/v-1/u)} & \text{if } 1 \leq u \leq 2. \end{cases}$$

*In particular,*

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \asymp n^{1/2+\lambda(\Pi_\gamma, u, v)}.$$

Here,  $\mathcal{S}_u^n$  denotes the space of all linear operators  $T : \ell_2^n \rightarrow \ell_2^n$  endowed with the norm  $\|T\|_{\mathcal{S}_u^n} := \|(s_k(T))_{k=1}^n\|_{\ell_u^n}$ , where  $(s_k(T))_{k=1}^n$  is the sequence of singular numbers of  $T$ . Besides the interpolation argument, our proof uses the close relationship between the Gaussian-summing norm of the identity operator  $\text{id}_E$  and the Dvoretzky dimension of a finite-dimensional Banach space  $E$  due to Pisier.

## 2 Complex interpolation of $B$ -summing operators

Our main tool will be an “interpolation theorem” for the  $B$ -summing norm of a fixed operator acting between finite-dimensional complex interpolation spaces; a similar approach for the  $(s, 2)$ -summing norm was used in [DM98] to study the well-known “Bennett–Carl inequalities” within the context of interpolation theory.

For all information on complex interpolation we refer to [BL78]. Given an interpolation couple  $[E_0, E_1]$  of complex Banach spaces and  $0 < \theta < 1$ , the associated complex interpolation space is

denoted by  $[E_0, E_1]_\theta$ . If  $E_0$  and  $E_1$  are finite-dimensional Banach spaces with the same dimensions, we speak of a finite-dimensional interpolation couple, and in this case we define

$$d_\theta[E_0, E_1] := \sup_m \|\mathcal{L}(\ell_2^m, [E_0, E_1]_\theta) \hookrightarrow [\mathcal{L}(\ell_2^m, E_0), \mathcal{L}(\ell_2^m, E_1)]_\theta\|.$$

Pisier [Pi90] and Kouba [Kou91] derived upper estimates for  $d_\theta[E_0, E_1]$  for particular situations (see also [DM99]); we will use the fact that for  $1 \leq p_0, p_1 \leq 2$

$$d_\theta[\ell_{p_0}^n, \ell_{p_1}^n] \leq \sqrt{2}; \quad (2.1)$$

in particular,  $\sup_n d_\theta[\ell_1^n, \ell_2^n] < \infty$ . Junge [Jun96, 4.2.6] gave an analogue for Schatten classes:

$$\sup_n d_\theta[\mathcal{S}_1^n, \mathcal{S}_2^n] < \infty. \quad (2.2)$$

These “uniform” estimates will be crucial for the applications of the following result:

**Proposition 3.** *Let  $0 < \theta < 1$ . Then for two finite-dimensional interpolation couples  $[E_0, E_1]$  and  $[F_0, F_1]$ , each  $T \in \mathcal{L}([E_0, E_1]_\theta, [F_0, F_1]_\theta)$  and each orthonormal system  $B \subset L_2(\mu)$*

$$\pi_B(T : [E_0, E_1]_\theta \rightarrow [F_0, F_1]_\theta) \leq d_\theta[E_0, E_1] \cdot \pi_B(T : E_0 \rightarrow F_0)^{1-\theta} \cdot \pi_B(T : E_1 \rightarrow F_1)^\theta.$$

*Proof.* For the moment set  $E_\theta := [E_0, E_1]_\theta$ ,  $F_\theta := [F_0, F_1]_\theta$ , and consider for  $\eta = 0, \theta, 1$  and  $\mathcal{F} = \{b_1, \dots, b_m\} \subset B$  the mapping

$$\begin{aligned} \Phi_\eta^{m, \mathcal{F}} : \mathcal{L}(\ell_2^m, E_\eta) &\rightarrow L_2(\mu, F_\eta) \\ S &\mapsto \sum_{i=1}^m b_i \cdot T S e_i. \end{aligned}$$

Since for each  $S = \sum_{j=1}^m e_j \otimes x_j \in \mathcal{L}(\ell_2^m, E_\eta)$

$$\|S\| = \sup_{x' \in B_{E'_\eta}} \left( \sum_{j=1}^m |\langle x', x_j \rangle|^2 \right)^{1/2},$$

we obviously get that

$$\pi_B(T : E_\eta \rightarrow F_\eta) = \sup\{\|\Phi_\eta^{m, \mathcal{F}}\| \mid m \in \mathbb{N}, \mathcal{F} \subset B \text{ with } |\mathcal{F}| = m\}.$$

For the interpolated mapping

$$[\Phi_0^{m, \mathcal{F}}, \Phi_1^{m, \mathcal{F}}]_\theta : [\mathcal{L}(\ell_2^m, E_0), \mathcal{L}(\ell_2^m, E_1)]_\theta \rightarrow [L_2(\mu, F_0), L_2(\mu, F_1)]_\theta$$

by the usual interpolation theorem

$$\|[\Phi_0^{m, \mathcal{F}}, \Phi_1^{m, \mathcal{F}}]_\theta\| \leq \|\Phi_0^{m, \mathcal{F}}\|^{1-\theta} \cdot \|\Phi_1^{m, \mathcal{F}}\|^\theta.$$

Since  $[L_2(\mu, F_0), L_2(\mu, F_1)]_\theta = L_2(\mu, [F_0, F_1]_\theta)$  (isometrically, see [BL78, 5.1.2]) we obtain

$$\|\Phi_\theta^{m, \mathcal{F}}\| \leq \|\mathcal{L}(\ell_2^m, [E_0, E_1]_\theta) \hookrightarrow [\mathcal{L}(\ell_2^m, E_0), \mathcal{L}(\ell_2^m, E_1)]_\theta\| \cdot \|[\Phi_0^{m, \mathcal{F}}, \Phi_1^{m, \mathcal{F}}]_\theta\|.$$

Consequently

$$\begin{aligned} \pi_B(T : [E_0, E_1]_\theta \rightarrow [F_0, F_1]_\theta) &= \sup\{\|\Phi_\theta^{m, \mathcal{F}}\| \mid m \in \mathbb{N}, \mathcal{F} \subset B \text{ with } |\mathcal{F}| = m\} \\ &\leq \sup\{d_\theta[E_0, E_1] \cdot \|\Phi_0^{m, \mathcal{F}}\|^{1-\theta} \cdot \|\Phi_1^{m, \mathcal{F}}\|^\theta \mid m \in \mathbb{N}, \mathcal{F} \subset B \text{ with } |\mathcal{F}| = m\} \\ &\leq d_\theta[E_0, E_1] \cdot \pi_B(T : E_0 \rightarrow F_0)^{1-\theta} \cdot \pi_B(T : E_1 \rightarrow F_1)^\theta, \end{aligned}$$

the desired result.  $\square$

Since for  $0 < \theta < 1$  and  $1 \leq p_0, p_1, p_\theta \leq \infty$  such that  $1/p_\theta = (1-\theta)/p_0 + \theta/p_1$  it holds  $[\ell_{p_0}^n, \ell_{p_1}^n]_\theta = \ell_{p_\theta}^n$  isometrically (see [BL78, 5.1.1]), we obtain together with (2.1) the following corollary:

**Corollary 4.** For  $0 < \theta < 1$  let  $1 \leq u_0, u_1, u_\theta \leq 2$  and  $1 \leq v_0, v_1, v_\theta \leq \infty$  such that  $1/u_\theta = (1 - \theta)/u_0 + \theta/u_1$  and  $1/v_\theta = (1 - \theta)/v_0 + \theta/v_1$ . Then

$$\lambda(\Pi_B, u_\theta, v_\theta) \leq (1 - \theta) \cdot \lambda(\Pi_B, u_0, v_0) + \theta \cdot \lambda(\Pi_B, u_1, v_1).$$

### 3 The proof of Theorem 1

As a generalization of the notion of  $\Lambda(p)$ -sets, an orthonormal system  $B \subset L_2(\mu)$  is said to be a  $\Lambda(p)$ -system if  $B \subset L_p(\mu)$  and there exists a constant  $c > 0$  such that for all  $f \in \text{span} B$  we have  $\|f\|_{L_p(\mu)} \leq c \cdot \|f\|_{L_2(\mu)}$ ; the infimum over all such constants  $c$  is denoted by  $K_p(B)$ . Now one direction of the equivalence in Theorem 1 can be formulated for general orthonormal systems:

**Proposition 5.** Let  $B \subset L_2(\mu)$  be a  $\Lambda(p)$ -system for all  $2 < p < \infty$ . Then for all  $1 \leq u, v \leq \infty$

$$\lambda(\Pi_B, u, v) = \lambda(\Pi_\gamma, u, v) = \begin{cases} 1/v & \text{if } 2 \leq u \leq \infty, \\ \max(0, 1/2 + 1/v - 1/u) & \text{if } 1 \leq u \leq 2. \end{cases}$$

*Proof.* Although the limit order of  $\Pi_\gamma$  is already known by the results of [LP74], the following proof may also be used to compute it independently (at least the upper estimates; the lower ones are somehow simple), but for simplicity we fall back upon this knowledge.

Since  $\Pi_2 \subset \Pi_B \subset \Pi_\gamma$  (see [PW98, 4.15];  $\Pi_2$  denotes the Banach operator ideal of all 2-summing operators), we only have to show that  $\lambda(\Pi_B, u, v) \leq \lambda(\Pi_\gamma, u, v)$ , and moreover, we conclude that  $\lambda(\Pi_B, u, v) \leq \lambda(\Pi_2, u, v) = \lambda(\Pi_\gamma, u, v)$  for all  $1 \leq u \leq \infty$  and  $1 \leq v \leq 2$ . For  $2 < v < \infty$  and  $2 \leq u \leq \infty$  it can be easily seen that

$$\Pi_B(\ell_u^m \hookrightarrow \ell_v^m) \leq K_v(B) \cdot m^{1/v}$$

(just copy the proof of [PW98, 3.11.11]), hence—together with the continuity of the limit order, see [Pie80, 14.4.8]—we obtain

$$\lambda(\Pi_B, u, v) \leq 1/v = \lambda(\Pi_\gamma, u, v)$$

for all  $2 \leq u, v \leq \infty$ . Now the case  $1 \leq u \leq 2 \leq v \leq \infty$  follows from Corollary 4: For  $1 \leq u \leq 2$  choose  $u_0 := 1$ ,  $u_1 := 2$ ,  $v_0 := 2$ ,  $v_1 := \infty$ ,  $\theta := 2/u'$  and  $u_u$  such that  $1/v_u = 1/u - 1/2$ . Then

$$\lambda(\Pi_B, u, v_u) \leq (1 - \theta) \cdot \lambda(\Pi_B, 1, 2) + \theta \cdot \lambda(\Pi_B, 2, \infty) = 0.$$

For arbitrary  $1 \leq u \leq 2 \leq v \leq \infty$  factorize through  $\ell_{v_u}^m$ :

$$\pi_B(\ell_u^m \hookrightarrow \ell_v^m) \leq m^{\max(0, 1/v + 1/2 - 1/u)} \cdot \pi_B(\ell_u^m \hookrightarrow \ell_{v_u}^m),$$

hence  $\lambda(\Pi_B, u, v) \leq \max(0, 1/v + 1/2 - 1/u) = \lambda(\Pi_\gamma, u, v)$ .  $\square$

The reverse implication in Theorem 1 follows from [Bau97]: (b) trivially implies  $\lambda(\Pi_B, \infty, \infty) = 0$ , and the comments after [Bau97, 7.12] then tell us that  $\Pi_p \subset \Pi_\Lambda$  for all  $2 < p < \infty$ . This in turn gives by [Bau97, 9.6] (see also [Bau99, 5.1]) that  $\Lambda$  is a  $\Lambda(p)$ -set for all  $2 < p < \infty$ .

Note that the last argument requires the setting of characters on a compact abelian group; Baur has recently informed us that her results are also valid for the non-abelian case, and therefore our Theorem 1 as well.

## 4 The proof of Theorem 2

*Proof.* For  $1 \leq v \leq 2$  by [TJ74]  $\mathcal{S}_v$  is of cotype 2, hence

$$n^{1/v + \min(1/2, 1-1/u)} = \pi_2(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \geq \pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \geq \mathbf{C}_2(\mathcal{S}_v)^{-1} \cdot \pi_2(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n)$$

(see e.g. [DM98, Corollary 3]) for  $1 \leq u \leq \infty$  and  $1 \leq v \leq 2$ ; here  $\mathbf{C}_2(\mathcal{S}_v)$  denotes the Gaussian cotype 2 constant of  $\mathcal{S}_v$ . We are left with the case  $2 \leq v \leq \infty$ ; first let  $u = v = \infty$ . Then by [PW98, 4.15.18] (a result of Pisier, see also [Pi89, 4.4] and [Pi86]) and [FLM77, 3.3] for each  $\varepsilon > 0$

$$\pi_\gamma(\mathcal{S}_\infty^n \hookrightarrow \mathcal{S}_\infty^n) \asymp \sqrt{D(\mathcal{S}_\infty^n, \varepsilon)} \asymp n^{1/2},$$

where  $D(X, \varepsilon)$  denotes the Dvoretzky dimension of a Banach space  $X$ , i.e. the largest  $m$  such that there exists an  $m$ -dimensional subspace  $X_m$  of  $X$  with Banach–Mazur distance  $d(X_m, \ell_2^m) \leq 1 + \varepsilon$  (see [PW98, 4.15.15]). Now the general case  $2 \leq u, v \leq \infty$  follows by factorization:

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \leq n^{1/v} \cdot \pi_\gamma(\mathcal{S}_\infty^n \hookrightarrow \mathcal{S}_\infty^n) \asymp n^{1/v+1/2},$$

and conversely

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \geq n^{1/v-1/2} \cdot \pi_\gamma(\mathcal{S}_2^n \hookrightarrow \mathcal{S}_2^n) = n^{1/v+1/2}.$$

The case  $1 \leq u \leq 2 \leq v \leq \infty$  is done by interpolation: We have (recall that  $\pi_2(\mathcal{S}_1^n \hookrightarrow \mathcal{S}_2^n) = n^{1/2}$ )

$$\pi_\gamma(\mathcal{S}_1^n \hookrightarrow \mathcal{S}_2^n) \asymp \pi_\gamma(\mathcal{S}_2^n \hookrightarrow \mathcal{S}_\infty^n) \asymp n^{1/2},$$

hence for  $1 < u < 2 < v_u < \infty$  and  $0 < \theta < 1$  such that  $1/v_u = 1/u - 1/2$  and  $\theta = 2/u'$

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_{v_u}^n) \leq d_\theta[\mathcal{S}_1^n, \mathcal{S}_2^n] \cdot \pi_\gamma(\mathcal{S}_1^n \hookrightarrow \mathcal{S}_2^n)^{1-\theta} \cdot \pi_\gamma(\mathcal{S}_2^n \hookrightarrow \mathcal{S}_\infty^n)^\theta \asymp n^{1/2};$$

recall that  $\sup_n d_\theta[\mathcal{S}_1^n, \mathcal{S}_2^n] < \infty$  by (2.2), and that  $[\mathcal{S}_1^n, \mathcal{S}_2^n]_\theta = \mathcal{S}_u^n$  and  $[\mathcal{S}_2^n, \mathcal{S}_\infty^n]_\theta = \mathcal{S}_{v_u}^n$  hold isometrically (this can be deduced from e.g. [PT68, Satz 8] and the complex reiteration theorem [BL78, 4.6.1]). The remaining estimates now follow easily from

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \geq \pi_\gamma(\ell_2^n \hookrightarrow \ell_2^n) = n^{1/2},$$

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \geq \pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_2^n) \cdot \|\mathcal{S}_v^n \hookrightarrow \mathcal{S}_2^n\|$$

and

$$\pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_v^n) \leq \pi_\gamma(\mathcal{S}_u^n \hookrightarrow \mathcal{S}_{v_u}^n) \cdot \|\mathcal{S}_{v_u}^n \hookrightarrow \mathcal{S}_v^n\|.$$

□

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## References

- [Bau97] F. Baur, *Banach operator ideals generated by orthonormal systems*, Univ. Zürich, Thesis, 1997.
- [Bau99] F. Baur, *Operator ideals, orthonormal systems and lacunary sets*, Math. Nachr. **197** (1999), 19–28.
- [BL78] J. Bergh and J. Löfström, *Interpolation spaces*, Springer-Verlag, 1978.

- [DM98] A. Defant and C. Michels, *Bennett–Carl inequalities for symmetric Banach sequence spaces and unitary ideals*, submitted (1998).
- [DM99] A. Defant and C. Michels, *A complex interpolation formula for tensor products of vector-valued Banach function spaces*, submitted (1999).
- [DJT95] J. Diestel, H. Jarchow, and A. Tonge, *Absolutely summing operators*, Cambridge Univ. Press, 1995.
- [FLM77] T. Figiel, J. Lindenstrauss, and V. D. Milman, *The dimension of almost spherical sections of convex bodies*, Acta Math. **139** (1977), 53–94.
- [Jun96] M. Junge, *Factorization theory for spaces of operators*, Univ. Kiel, Habilitationsschrift, 1996. Currently available on the net under <http://www-computerlabor.math.uni-kiel.de/~mjunge/preprints.html>
- [Kou91] O. Kouba, *On the interpolation of injective or projective tensor products of Banach spaces*, J. Funct. Anal. **96** (1991), 38–61.
- [LP74] W. Linde and A. Pietsch, *Mappings of Gaussian cylindrical measures in Banach spaces*, Theory Prob. Appl. **19** (1974), 445–460.
- [LT79] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces II: Function spaces*, Springer-Verlag, 1979.
- [LR75] J. López and K. Ross, *Sidon sets*, Marcel Dekker, inc., 1975.
- [Pie80] A. Pietsch, *Operator ideals*, North-Holland, 1980.
- [PT68] A. Pietsch and H. Triebel, *Interpolationstheorie für Banachideale von beschränkten linearen Operatoren*, Studia Math. **31** (1968), 95–109.
- [PW98] A. Pietsch and J. Wenzel, *Orthonormal systems and Banach space geometry*, Cambridge Univ. Press, 1998.
- [Pi78] G. Pisier, *Topics on Grothendieck’s theorem*, Proceedings of the international conference on operator algebras, ideals, and their applications in theoretical physics, Teubner (1978), 44–57.
- [Pi86] G. Pisier, *Probabilistic methods in the geometry of Banach spaces*, Springer Lecture Notes in Math. **1206** (1986), 167–241.
- [Pi89] G. Pisier, *The volume of convex bodies and Banach space geometry*, Cambridge Univ. Press, 1989.
- [Pi90] G. Pisier, *A remark on  $\Pi_2(\ell_p, \ell_p)$* , Math. Nachr. **148** (1990), 243–245.
- [Sei95] J. Seigner, *Über eine Klasse von Idealnormen, die mit Orthonormalsystemen gebildet sind*, Friedrich-Schiller-Univ. Jena, Thesis, 1995.
- [TJ74] N. Tomczak-Jaegermann, *The moduli of smoothness and convexity and the Rademacher averages of trace classes  $S_p$  ( $1 \leq p < \infty$ )*, Studia Math. **50** (1974), 163–182.
- [TJ89] N. Tomczak-Jaegermann, *Banach–Mazur distances and finite-dimensional operator ideals*, Longman Scientific & Technical, 1989.